# General theories of logical systems

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*Ústav informatiky AV CR*[1]

*LS 2013/2014*
*čas přednášky bude stanoven emailovou úmluvou*[2]

Algebraic logic is the branch of mathematical logic that studies logical systems by giving them algebraic semantics. It mainly capitalizes on the standard Linbenbaum–Tarski proof of completeness of classical logic w.r.t. the two-element Boolean algebra, which can be analogously repeated in other logical systems yielding completeness w.r.t. other kinds of algebras. *Abstract* algebraic logic (AAL) determines what are the essential elements in these proofs and develops an abstract theory of the possible ways in which logical systems can be related to an algebraic counterpart. The usefulness of these methods is witnessed by the fact that the study of many logics, relevant for mathematics, computer science, linguistics or philosophical purposes, has greatly benefited from the algebraic approach, that allows to understand their properties in terms of equivalent algebraic properties of their semantics.

This course is a self-contained introduction to AAL. We start from the very basics of AAL, develop its general and systematical theory and illustrate the results with applications to particular examples of propositional logics.

1. Basic notions of algebraic logic: formulae, proofs, logical matrices, filters, closure operators, closure systems, Schmidt Theorem, abstract Lindenbaum Lemma. Completeness theorem w.r.t. the class of all models. (Weakly) implicative logics. Examples on substructural and fuzzy logics.
2. Lindenbaum–Tarski method for weakly implicative logics: Leibniz congruence, reduced matrices, and completeness theorem w.r.t. the class of reduced models. Operators on classes of matrices. Relatively (finitely) subdirectly irreducible matrices (RFSI). Completeness theorem w.r.t. RFSI reduced models. Algebraizability and order algebraizability. Examples on substructural and fuzzy logics.
3. Leibniz operator on arbitrary logics. Leibniz hierarchy: protoalgebraic, equivalential and (weakly) algebraizable logics. Regularity and finiteness conditions. Alternative characterizations of the classes in the hierarchy.
4. Bridge theorems (deduction theorems, Craig interpolation, Beth definability). Generalized disjunctions and proof by cases properties and their role in AAL.

## Study literature

[1] Willem J. Blok and Don L. Pigozzi. *Algebraizable Logics*, Memoirs of the American Mathematical Society, vol. 396, Providence, RI, USA, 1989, freely downloadable from `http://orion.math.iastate.edu/dpigozzi/`.

[2] Petr Cintula and Carles Noguera. A General Framework for Mathematical Fuzzy Logic, *Handbook of Mathematical Fuzzy Logic - Volume 1*, Studies in Logic, Mathematical Logic and Foundations, vol. 37, Petr Cintula and Petr Hájek and Carles Noguera (eds.), pp. 103–207, College Publications, London, 2011. Downloadable from `http://www.carlesnoguera.cat`.

[3] Janusz Czelakowski. *Protoalgebraic Logics*, vol. 10 of *Trends in Logic*. Kluwer, 2001.

[4] Josep Maria Font, Ramon Jansana, and Don Pigozzi. A survey of Abstract Algebraic Logic. *Studia Logica*, 74(1–2):13–97, 2003.

[1] Pod Vodárenskou věží 2, poblíž stanice metra Ládví, `www.cs.cas.cz`.

[2] Zájemci o účast mohou poslat své časové možnosti do 16.2.2014 na email `cintula@cs.cas.cz`.