Logic of Questions from the Viewpoint of Dynamic Epistemic Logic

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1 Introduction

Questions play an important role in communication in both natural language and artificial languages. However, the logic of questions—the discipline of which the aim is to represent questions as logical entities and to explore relations between these entities—has been receiving relatively little attention in literature until quite recently.

The logic of questions has, maybe surprisingly, a long history (cf. (Harrah, 2002)). F. Cohen and R. Carnap seem to be the first authors attempting to formalize questions in a logical framework—their attempts date back to the 1920s. The first 'boom' of logical approach to questions took place in the 1950s (Hamblin, Prior, Stahl) and continued in the 1960s (Åqvist, Hintikka, Kubiński). The first comprehensive monograph on questions was published in the 1970s (Belnap & Steel, 1976). The 1990's gave birth to several approaches, some of which are still influential. The most important are the semantic approach of Jeroen Groenendijk and Martin Stokhof and Inferential Erotetic Logic of Andrzej Wiśniewski. Recently the logic of questions receives more attention in connection with dynamic logic and game theory (e.g., (Benthem & Minică, 2009) and (Genot, 2009)).

Our main goal is to consider questions in the process of communication. When an agent asks a question, she does not only provide some facts to her listeners, but she also reveals the structure of her knowledge, in particular, what she presupposes and what she considers as possible updates of her current knowledge. We see asking a question as a process of information exchange, which can be quite complex. From a logician's point of view the most appropriate framework for representing this process is the one of dynamic epistemic logic. The machinery of this framework allows for the representation of information states of individual agents and their changes in the process of receiving information from the other agents. We focus on the case when new information is 'public', i.e., available to all agents in a particular group, typically the new information is announced so that

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everybody in the group can hear it. This very situation serves also as a motivation of the public announcement logic (see (Ditmarsch, Hoek, & Kooi, 2008)), which we shall employ.

2 Public announcement logic

Public announcement logic in the sense of (Ditmarsch et al., 2008) is an extension of the multi-agent epistemic logic S5, which is a multi-modal logic, where each of the finitely many modal operators is an S5-modality.

We start with a propositional multimodal language $\mathcal{L}^K$—a classical propositional language $\mathcal{L}$ with finitely many modalities K_i to be read as 'the agent i knows ...'. The formulas of $\mathcal{L}^K$ are defined in a standard way ($p, q, \ldots$ are signs for atomic formulas):

$$\varphi ::= p \mid \neg\psi \mid \psi_1 \vee \psi_2 \mid \psi_1 \wedge \psi_2 \mid \psi_1 \rightarrow \psi_2 \mid \psi_1 \leftrightarrow \psi_2 \mid K_i\psi$$

As in the standard modal logics we can define a dual operator to K_i—an epistemic possibility M_i ('the agent i admits that possibly ...'): $M_i\varphi \equiv \neg K_i\neg\varphi$.

The semantics of $\mathcal{L}^K$ is based on the standard Kripke-style models. A *Kripke frame* is a relational structure $\mathcal{F} = \langle S, R_1, \ldots, R_m\rangle$ with a set of states (points, indices, possible worlds) S and accessibility relations $R_i \subseteq S^2$, for agents $i \in \{1, \ldots, m\}$. *Kripke model* $\mathbf{M}$ is a pair $\langle \mathcal{F}, v\rangle$ where v is a valuation of atomic formulas. The satisfaction relation $\models$ is defined in a standard way:

- $(\mathbf{M}, s) \models p$ iff $(\mathbf{M}, s) \in v(p)$
- $(\mathbf{M}, s) \models \neg\varphi$ iff $(\mathbf{M}, s) \not\models \varphi$
- $(\mathbf{M}, s) \models \psi_1 \vee \psi_2$ iff $(\mathbf{M}, s) \models \psi_1$ or $(\mathbf{M}, s) \models \psi_2$
- $(\mathbf{M}, s) \models \psi_1 \wedge \psi_2$ iff $(\mathbf{M}, s) \models \psi_1$ and $(\mathbf{M}, s) \models \psi_2$
- $(\mathbf{M}, s) \models \psi_1 \rightarrow \psi_2$ iff $(\mathbf{M}, s) \models \psi_1$ implies $(\mathbf{M}, s) \models \psi_2$
- $(\mathbf{M}, s) \models K_i\varphi$ iff $(\mathbf{M}, s_1) \models \varphi$, for each s_1 such that sR_is_1

Each R_i is an equivalence relation on S^2.

On top of the individual epistemic modalities K_i and M_i we introduce group modalities E_G and C_G. $E_G\varphi$ means 'each agent from $G \subseteq \{1, \ldots, m\}$ knows φ', i.e.,

$$E_G\varphi \leftrightarrow \bigwedge_{i \in G} K_i\varphi$$

We shall call E_G *group knowledge.* Let us stress, that E_G does not guarantee that a member of the group G knows that she shares the same information with some other members of the group.

The second group modality C_G is stronger, $C_G\varphi$ requires not only that φ is group knowledge, but also that this fact is reflected by everybody in the group G

(everybody knows that everybody knows φ and everybody knows that everybody knows that everybody knows φ, etc.). We can see C_G as an infinite conjunction of all finite iterations of the group knowledge E_G:

$$C_G\varphi \leftrightarrow \varphi \wedge E_G\varphi \wedge E_GE_G\varphi \wedge E_GE_GE_G\varphi \wedge \ldots$$

This knowledge, called *common knowledge*, is maximally shared in the sense that everybody from G is aware of it. Such type of knowledge is essential for collective behavior and coordination of collective actions.

As we said E_G is definable in the language $\mathcal{L}^K$, so adding group knowledge is just a conservative extension of the background multimodal epistemic logic. However, this is not the case of common knowledge. Multimodal epistemic logic with common knowledge is a stronger system $\mathcal{L}^{KC}$ which we obtain from $\mathcal{L}^K$ adding the operator C_G and the clause:

- $(\mathbf{M}, s) \models C_G\varphi$ iff $(\mathbf{M}, s_1) \models \varphi$ for each s_1 such that $s \left(\bigcup_{i\in G} R_i\right)^* s_1$

$\left(\bigcup_{i\in G} R_i\right)^*$ is a reflexive and transitive closure of $\bigcup_{i\in G} R_i$ and it means that s_1 is accessible from s by each R_i ($i \in G$) in k steps, for any $k \geq 0$. Common knowledge is stronger than group knowledge:

$$C_G\varphi \to E_G\varphi \to K_i\varphi$$

is valid for each $i \in G$.

2.1 Public announcement

Let us imagine a group of three players: Ann, Bill, and Catherine. Each of them has one card and nobody can see the cards of the others. One of the cards is the Joker and everybody knows this fact. Ann received the Joker, but neither Bill nor Catherine know which of the other two players, has it. In particular, both of them are not able to distinguish between the states where Ann has the Joker and where she has not. If Ann publicly announces

> “I've got the Joker.”,

everybody in the group learns this fact. Situations where Ann does not have the Joker are excluded from the (epistemic) models of Bill and Catherine.

Our example gives a typical situation represented in the public announcement logic—after a public announcement of a statement φ (“I've got the Joker”), some other statement ψ holds (e.g., “Bill knows Ann has the Joker and Catherine knows Ann has the Joker”). In fact the author of an announced statement is irrelevant in our framework. The statement is understood as information coming to each member of a group in the same way. From this point of view Ann's announcement in our example has the same effect as if an external observer announces “Ann has the Joker”.

Formally we introduce the logic of public announcement as an extension of the system $\mathcal{L}^{KC}$. We define a box-like operator [ ], such that the intended meaning of $[\varphi]\psi$ is 'after the public announcement of φ, it holds that ψ'. The semantics of the new announcement operator is:

- $(\mathbf{M}, s) \models [\varphi]\psi$ iff $(\mathbf{M}, s) \models \varphi$ implies $(\mathbf{M}|_\varphi, s) \models \psi$

where $\mathbf{M}|_\varphi = \langle\langle S', R'_1, \ldots, R'_m\rangle, v'\rangle$ is defined as follows:

$$\begin{aligned} S' &= \{s \in S \mid s \models \varphi\} \\ R'_i &= R_i \cap S'^2 \\ v'(p) &= v(p) \cap S' \end{aligned}$$

The model $\mathbf{M}|_\varphi$ is obtained from $\mathbf{M}$ by deleting of all states where φ is not true and by the corresponding restrictions of accessibility relations and the valuation function. Again we can introduce a dual operator $\langle\ \rangle$ defined in a standard way as $\langle\varphi\rangle\psi$ iff $\neg[\varphi]\neg\psi$. If we rewrite the corresponding semantic clause, we obtain

- $(\mathbf{M}, s) \models \langle\varphi\rangle\psi$ iff $(\mathbf{M}, s) \models \varphi$ and $(\mathbf{M}|_\varphi, s) \models \psi$

The intended meaning of the dual operator is 'after a *truthful* announcement of φ, it holds that ψ'. It is easy to see that the diamond-like operator is stronger:

Lemma 1. $\models \langle\varphi\rangle\psi \to [\varphi]\psi$

The following proposition shows that languages $\mathcal{L}^{K[]}$ and $\mathcal{L}^{K}$ have the same expressive power—it provides a reduction of formulas with the public announcement operator to the epistemic ones. The corresponding equivalences in the proposition give in fact an axiomatization of the announcement operator in the public announcement epistemic logic without common knowledge (Ditmarsch et al., 2008, p. 81).

Proposition 1. *The following equivalences are valid (*$\circ \in \{\wedge, \vee, \to\}$*):*

$$\begin{aligned} [\varphi]p &\leftrightarrow (\varphi \to p) \\ [\varphi]\neg\psi &\leftrightarrow (\varphi \to \neg[\varphi]\psi) \\ [\varphi](\psi \circ \chi) &\leftrightarrow ([\varphi]\psi \circ [\varphi]\chi) \\ [\varphi]K_i\psi &\leftrightarrow (\varphi \to K_i[\varphi]\psi) \\ [\varphi][\psi]\chi &\leftrightarrow [\varphi \wedge [\varphi]\psi]\chi \end{aligned}$$

For common knowledge there is no such reduction, the language $\mathcal{L}^{KC[]}$ is more expressive than $\mathcal{L}^{KC}$. We only have a rule describing the relationship between the public announcement and common knowledge, e.g., the one in (Ditmarsch et al., 2008, p. 83):

$$\frac{(\chi \wedge \varphi) \to [\varphi]\psi \wedge E_G\chi}{(\chi \wedge \varphi) \to [\varphi]C_G\psi} \tag{1}$$

2.2 Updates

Let us return to our example. As we said, members of a group learn what was announced. In particular, if Ann says

> "I've got the Joker.",

the announced fact becomes commonly known in the group of players. This seems to suggest that a publicly announced proposition becomes common knowledge . But what if Ann says:

> "You don't know it yet, but I've got the Joker."

Although the formula $(J_a \wedge \neg K_b(J_a) \wedge \neg K_c(J_a))$ is true in the moment of announcement, it is evident that its epistemic part ("you don't know it yet") becomes invalid after it is announced. So the formula $(J_a \wedge \neg K_b(J_a) \wedge \neg K_c(J_a))$ becomes false after the announcement. Formulas, which became false after they are truthfully announced (as in our example), are in public announcement logic called an *unsuccessful update*; if they are true, we call them a *successful update.* For the [ ] modality we have similar notions. If a formula $[\varphi]\varphi$ is valid, we call it a *successful formula*, otherwise it is an *unsuccessful formula.*

Definition 1.

- *Formula* φ *is a* successful update *in* $(\mathbf{M}, s)$ *iff* $(\mathbf{M}, s) \models \langle\varphi\rangle\varphi$.
- *Formula* φ *is an* unsuccessful update *in* $(\mathbf{M}, s)$ *iff* $(\mathbf{M}, s) \models \langle\varphi\rangle\neg\varphi$.
- *Formula* φ *is a* successful formula *iff* $[\varphi]\varphi$ *is valid, otherwise it is an* unsuccessful formula

If a formula is an unsuccessful update, it cannot be commonly known in the updated model. Using the soundness proof of the rule (1) we can prove that a formula is true after a public announcement if and only if it gets common knowledge after the announcement (see (Ditmarsch et al., 2008, p. 83 and 86)).

Proposition 2. $[\varphi]\psi$ *is valid iff* $[\varphi]C_G\psi$ *is valid.*

As a consequence we get

Proposition 3. $[\varphi]\varphi$ *is valid iff* $[\varphi]C_G\varphi$ *is valid.*

Thus, publicly announced successful formulas are commonly known.[1]

3 Questions

Let us return to our card players. It seems reasonable for Catherine to say:

> "Who has got the Joker?"

[1] Atoms, $K_i\varphi$, and $\neg K_i\varphi$ (for every φ) are examples of successful formulas.

We recognize this sentence as an *interrogative sentence* because of its word order and the question mark. In speech, we recognize uttering a question because of the intonation and interrogative pronunciation. The *interrogative speech act* is important in the pragmatic approach to questions, where we focus on the fact that a questioner expresses a request to an addressee.

In logic of questions it is generally accepted that interrogative sentences have a different meaning than indicative ones.[2] In our approach to questions we pay special attention to their semantic content. The traditional starting point in the semantics of questions are Hamblin's postulates:

1. An answer to a question is a statement.
2. Knowing what counts as an answer is equivalent to knowing the question.
3. The possible answers to a question are an exhaustive set of mutually exclusive possibilities.

These postulates have been subjected to some criticism, see (Harrah, 2002) and (Groenendijk & Stokhof, 1997). We are not going to join these critical discussions. In our project we consider 'Hamblin's picture' as partially adequate to the formalization of questions in the epistemic framework. The first two postulates seem to be the first step towards the formalization of questions known as *set-of-answers methodology* (SAM, for short). We implement them in our framework using a modification of Wiśniewski's SAM (see, e.g., (Wiśniewski, 1995, 2001) and (Peliš, 2008)) where questions are represented as finite sets of formulas.[3] We shall not in general accept the third postulate, we shall just use it for delineating a special subclass of questions in section 3.1.

We extend the language of epistemic logic $\mathcal{L}^{KC}$ adding brackets $\{,\}$ and the question mark $?_i$ for a question of an agent i. We obtain the language $\mathcal{L}_Q^{KC}$. A question is any formula of the form

$$?_i\{\alpha_1, \ldots, \alpha_n\},$$

such that $\alpha_1, \ldots, \alpha_n$ are syntactically distinct formulas of our extended epistemic language $\mathcal{L}_Q^{KC}$ and $n \geq 2$ (there are at least two answers). We shall denote a question of an agent i as $Q^i, Q^i_1, \ldots$, etc. As an index we can also use a subset of the set of agents. The intended reading of a question Q^i is:

> 'Is it the case that α_1 or is it the case that α_2 ... or is it the case that α_n?'

We shall call $\{\alpha_1, \ldots, \alpha_n\}$ the set of *direct answers* to the question Q^i and denote it dQ^i.

Let us assume that Catherine from our example asks the question:

[2] There are exceptions, however, some logicians, e.g., Pavel Tichý hold the view that the logical form of both interrogatives and indicatives are the same, what is different is the pragmatic aspect (Tichý, 1978).

[3] Unlike Wiśniewski, we allow the mixing of declarative and interrogative formulas together, so we can have questions about questions as well.

"Who has got the Joker: Bill, or Ann?"

This question has the following direct answers:

- Bill has got the Joker.
- Ann has got the Joker.

Catherine expresses that she

1. does not know any of the answers to the question,
2. considers all the answers to be possible, and, moreover,
3. presupposes what is implicitly included in the answers, i.e., it must be the case that either Bill has got the Joker, or Ann has got it.

Catherine (the agent-questioner) provides the information of her ignorance (item 1) as well as the expected way to extend her knowledge; 2 says that there are two possibilities to make this extension and 3 that exactly one of the possibilities is expected to be true. A publicly asked question allows listeners to male a picture of the questioner's knowledge structure, which is an important part of communication and problem solving in groups.

In semantics for a majority logical systems we speak about the truth/falsity of a formula (in a particular state of a particular model). It is clear that it makes little sense to speak about the truth/falsity of a question. We introduce instead a concept of *askability* of a question; an askable question is a question which is in some sense 'reasonable' to ask in a certain situation. 'Reasonability' corresponds to the following three conditions:

1. **Non-triviality** It is not reasonable to ask a question if the answer is known.
2. **Admissibility** Each direct answer is considered as possible.
3. **Context** At least one of the direct answers must be the right one.

These askability conditions will play the role of the truth conditions in the formal definition of semantics of interrogative formulas.

Definition 2. *It holds for a question* $Q^i = ?_i\{\alpha_1, \dots, \alpha_n\}$ *that* $(\mathbf{M}, s) \models Q^i$ *iff*

1. $(\forall \alpha \in dQ^i)((\mathbf{M}, s) \not\models K_i\alpha)$
2. $(\forall \alpha \in dQ^i)((\mathbf{M}, s) \models M_i\alpha)$
3. $(\mathbf{M}, s) \models K_i\big(\bigvee_{\alpha_j \in dQ^i} \alpha_j\big)$

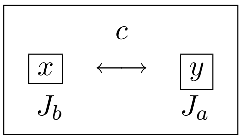
We say that Q^i *is* askable *in the state* $(\mathbf{M}, s)$ *(by an agent* i*).*

As we can see, the freedom in the syntactical form of questions was compensated by restrictions in their semantics. We say that a question is (generally) *askable* iff there is a model and a state where the question is askable (by an agent). Askable questions include neither contradiction nor tautology among their direct answers. The former is excluded by the second condition and the latter by the first one. If we work in systems extending classical logic, the first condition is equal to $(\mathbf{M}, s) \models \neg K_i\alpha$, i.e., $(\mathbf{M}, s) \models M_i\neg\alpha$, for each $\alpha \in dQ^i$. We can see the questioner as admitting the possibility of $\neg\alpha$ for each direct answer α to a question Q^i.

Catherine's question

> "Who has got the Joker: Bill, or Ann?"

can be formalized by $?_c\{J_b, J_a\}$. This question is askable in a state s whenever there are at least two indistinguishable states with J_b true in one of them and J_a true in the other.



Non-triviality condition says that Catherine is not able to distinguish between the states x and y, both formulas J_b and J_a are considered as possible (admissibility condition), and exactly one of the statements J_b and J_a must be the case (context condition). The shape of our model is influenced by more restrictions given by the context of our card-players example, i.e, commonly known rules of the game (in particular, there is just one Joker in the game).

3.1 Askability, answerhood, and safe questions

Askability is a complex term. We shall discuss now what happens if we violate the askability conditions. Whenever the non-triviality is violated, an agent knows at least one of the direct answers. Then she is able to answer a question. We say that the question is *answered* for her in a state. The violation of the admissibility condition brings about the situation where an agent does not consider at least one of the direct answers possible. She does not need to know any direct answer to a question, but she can reject some of them. Then we say that the question is *partially answered* for her in a state. The violation of the last condition (context) is of another kind. An agent is not aware of what is presupposed. Such a question is called *weakly presupposed* by the agent in a state. Let us sum up the introduced terms:

Definition 3. *Let Q^i be a question $?_i\{\alpha_1, \ldots, \alpha_n\}$.*

- Q^i *is* answered *in* $(\mathbf{M}, s)$ *(for an agent i) iff* $(\mathbf{M}, s) \models \bigvee_{\alpha_j \in dQ^i}(K_i\alpha_j)$. *We write* $(\mathbf{M}, s) \models A_iQ$.

- Q^i *is* partially answered *in* $(\mathbf{M}, s)$ *(for an agent i) iff* $(\mathbf{M}, s) \models \bigvee_{\alpha_j \in dQ^i}(K_i \neg\alpha_j)$. *We write* $(\mathbf{M}, s) \models P_i Q$.

- Q^i *is* weakly presupposed *in* $(\mathbf{M}, s)$ *(by an agent i) iff* $(\mathbf{M}, s) \not\models K_i\big(\bigvee_{\alpha_j \in dQ^i} \alpha_j\big)$.

The violation of the context condition means that the agent does not believe that at least one of $\alpha \in dQ$ is the right answer. We shall not discuss this condition as it is not essential for the purposes of this paper. We can avoid the context condition if we restrict ourselves to a special class of questions—*safe questions*, for which it is trivially satisfied. A question Q^i is *safe* if and only if $\big(\bigvee_{\alpha_j \in dQ^i} \alpha_j\big)$ is tautology. This class includes, among others, the well known yes-no questions, i.e., questions of the form $?_i\{\alpha, \neg\alpha\}$.[4]

Definition 4. *A question* Q^i *is (for an agent i)*

- safe in a state $(\mathbf{M}, s)$ *iff* $(\mathbf{M}, s_1) \models \big(\bigvee_{\alpha_j \in dQ^i} \alpha_j\big)$, *for each* s_1 *such that* $sR_i s_1$.

- safe *iff* $\big(\bigvee_{\alpha_j \in dQ^i} \alpha_j\big)$ *is valid.*

Another way around the context condition is to define explicitly a common context, which is shared by all members of the group. This is the case of our motivational example—the commonly known context enables us to see Catherine's question $?_c\{J_b, J_a\}$ as safe in the corresponding state.

Catherine's question is a question with mutually exclusive direct answers. We shall make use of a weaker condition—the one where a direct answer does not exclude any of the other direct answers but just some of them. We shall call it *questions with pairs of mutually exclusive direct answers.*[5] There is one more reason to restrict ourselves to the class of questions we just introduced. In general, it is not true that an answered question is also partially answered. $A_i Q$ implies $P_i Q$ only for questions with pairs of mutually exclusive direct answers.

The following proposition is proved in (Peliš, 2009).

Proposition 4. *Let* Q^i *be a safe question in* $(\mathbf{M}, s)$ *with pairs of mutually exclusive direct answers. Then the following conditions are equivalent:*

- $(\mathbf{M}, s) \models \neg Q^i$

- $(\mathbf{M}, s) \models P_i Q$

- *There is a formula* φ *such that* $(\mathbf{M}, s) \models A_i?\{\varphi, \neg\varphi\}$ *and* $Q^i \to ?_i\{\varphi, \neg\varphi\}$ *is valid.*

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[4] The term *safe question* originates from Belnap, the presented meaning is inspired by Wiśniewski.

[5] For example askable safe questions are of this kind. We require askability to avoid safe questions with tautological direct answers.

Partial answerhood of a question Q^i in some state is equivalent to the existence of a yes-no question, which is answered at that state and implied by Q^i. From the validity of $Q^i \rightarrow ?_i\{\varphi, \neg\varphi\}$ we know that inaskability of $?_i\{\varphi, \neg\varphi\}$[6] implies inaskability of Q^i and, therefore, φ (as well as $\neg\varphi$) imply either some $\alpha \in dQ^i$ or $\neg\alpha$ (for $\alpha \in dQ^i$).

4 Public announcement updates and questions

Let us put things together. We are interested in the behavior of questions in public announcement logic. If our background epistemic logic is multi-modal S5, then the formula $[Q^i]Q^i$ is valid, i.e.,

Fact 1. *Questions are successful formulas.*

In S5-models a question Q^i, which is askable in a state s, is askable in all states from the equivalence class sR_i. No 'cutting' of states in the model $\mathbf{M}$ forced by the public announcement of Q^i results in $(\mathbf{M}, s) \models Q^i$ and $(\mathbf{M}|_{Q^i}, s) \not\models Q^i$. Thus, a publicly announced question is commonly known (see Proposition 3). In other words there is no model and state such that $(\mathbf{M}, s) \not\models [Q^i]Q^i$.

Successful formulas have an important property: they do not bring anything new if they are announced repeatedly.

Fact 2. *Let φ be a successful formula. $[\varphi][\varphi]\psi \leftrightarrow [\varphi]\psi$ is valid.*

The proof is straightforward: $[\varphi][\varphi]\psi$ is equivalent to $[\varphi \wedge [\varphi]\varphi]\psi$ (Proposition 1), which is equivalent to $[\varphi]\psi$, because of the validity of $[\varphi]\varphi$ (φ is successful).

It is no surprise that askable questions (in a state) are successful updates.

Fact 3. $(\mathbf{M}, s) \models Q^i$ *iff* $(\mathbf{M}, s) \models \langle Q^i \rangle Q^i$.

Whenever an agent publicly asks a question, it does not cause any change in her epistemic model, it remains askable until she gets some new information.

The last point we are going to talk about is the relationship between public announcement and answerhood. Whenever a question is (partially) answerable in a state, then there is a formula φ such that after a public announcement of φ the question becomes inaskable there. Let us return to our group of players. Ann has the Joker. Neither Bill nor Catherine know it. If Catherine publicly asks

> "Who has got the Joker?",

Bill can infer: "I have not the Joker and Catherine does not know who has it, therefore Ann has it." Catherine's question was *informative* for Bill, it caused that the question "Who has got the Joker?", which was askable for Bill, became inaskable after Catherine asked it, even if her question was not (partially) answered. This leads us to the definition of *informative formula*.

[6] $(\mathbf{M}, s) \models \neg ?_i\{\varphi, \neg\varphi\}$ iff $(\mathbf{M}, s) \models A_i?\{\varphi, \neg\varphi\}$.

Definition 5. *A formula φ is* informative *(for an agent i) with respect to Q in* $(\mathbf{M}, s)$ *iff* $(\mathbf{M}, s) \models Q^i \wedge \langle\varphi\rangle\neg Q^i$.

Contrary to partial answerhood (see the commentary below Proposition 4) there need not be any logical connection between an informative formula and direct answers to the question. The informativeness can be forced by the shape of the particular model.

In a multiagent epistemic approach we can understand a question as a 'task' (or a 'problem') to be solved by a particular group of agents. Communication is one of the basic tools of a group searching for a solution to a problem (an answer to a question) and asking questions is one of the essential parts of this communication. Questions may not only be about a fact *Who has got the Joker?*, but also about knowledge *Do you know who has got the Joker, Catherine?* or about a question. For example, Bill has a problem (he does not know who has the Joker) and wants to know if Catherine has the same problem. So he can ask *Would you ask 'Who has got the Joker?', Catherine?*. In fact Bill asks Catherine whether the question *Who has got the Joker?* is a reasonable (askable) question for her:

$$?_b\{?_c\{J_b, J_a\}, \neg ?_c\{J_b, J_a\}\}$$

It is a yes-no question. The first direct answer means that Catherine would ask *Who has got the Joker?*, i.e., the question $?_c\{J_b, J_a\}$ is askable for her. The second direct answer means, this question is not askable for her, which according to Proposition 4 means that Catherine can (at least partially) answer that question.

5 Conclusion

We have presented the first step in the application of questions in the framework of dynamic epistemic logic. There are still many things to be done. In particular, we worked with the 'classical' public announcement logic based on the modal logic S5. We would like to use weaker systems of modal logic, so we will have to modify the definition of public announcement.

We would also like to study more deeply the relationship between group knowledge modalities and the problem of a search for an answer in a group of agents. We defined the askability and answerhood conditions for a single agent, but we can generalize this for a group of agents: it seems to be reasonable to require that a question is (partially) answered (for a group) if a (partial) answer is common knowledge (in this group). The dynamic aspect of common knowledge is mirrored in *relativized common knowledge*, see (Ditmarsch et al., 2008, section 7.8). Another interesting problem is under which conditions a group is able to find an answer to a question (without additional external information). This problem is closely connected to the notion of *implicit* (or *distributed*) knowledge.[7]

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[7] See, e.g., (Fagin, Halpern, Moses, & Vardi, 2003) for the corresponding definitions.

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